Supply chain configuration modeling based on colored Petri-nets

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Abstract - In nowadays global manufacturing, supply chain configuration modeling has been recognized as a linchpin to form different supply chains in response to changing customer demands. It is accomplished through addressing properly a high variety of supplies. This paper develops a set of formalism based on colored Petri-nets to model supply chain configuration with focus on product, process and logistics design. The Petri-net technique is applied to address such issues as generic variety representation, constraint satisfaction and flow control. Also reported is a case study of configuring supply chains for motor products in a multinational manufacturing company.

I. INTRODUCTION

Supply chain configuration (SCC) is inherently complex due to the dynamic, decentralized and distributed characteristics of a supply chain network of a manufacturer. Each node in the supply chain network often possesses several alternative options, which are autonomous organizations with unique resource, capability, objective, and competency [1]. Configuring a supply chain not only requires to decide alternative suppliers, delivery modes and inventory levels, but also to specify manufacturing processes, such as operations sequence, lead-times, setups, and manufacturing resources [2]. Due to the constant flux of changes resulted from global competition, there are many uncertain or random events in a supply chain, e.g., customer demand variations, delivery time alternations and production fluctuations. Moreover, a number of supply chain configurations can be obtained from a generic supply chain network according to the particular customer requirements and the specific operations environments. Therefore, it is imperative to model the process of configuring supply chains while handling a high variety of supplies and meeting the constraints of a supply chain.

This paper introduces a set of formalism based on colored Petri-nets (CPNs) for supply chain configuration and evaluation. The PN technique can deal with the issues regarding generic variety representation, constraints satisfaction and flow control [4]. The idea of modeling supply chain configuration is to explicitly grasp all supply-related information and activities, as well as their interdependences in conjunction with product, process and logistics decisions making.

The rest of the paper is organized as follows. Section 2 describes the problem context of supply chain configuration. The modeling formalism is introduced in Section 3. Section 4 reports an application of the proposed formalism in modeling supply chain configuration for motor products in a multinational manufacturing company. The conclusions are drawn in Section.

II. PROBLEM FORMULATION

An effective supply chain system necessitates the integration of product platform decisions, manufacturing process decisions and supply sourcing decisions. One crucial issue in designing such supply chains is to identify the key elements and their complex relationships in the marketing, product, process, and supply domains.

Definition 1: A supply chain network SCN is defined as a two tuple: SCN :=< SE, F >, where SE is a set of supply chain entities; F is a set of material flows.

Definition 2: The supply chain entity set SE is defined as a two tuple: SE :=< E^A, E^S >, where E^A is a set of assembly entities; E^S is a set of supplier entities.

SE = {e_i | ∀i = 1,...,m} consists of a set of supply chain entities and collaboratively produces a product family.

Definition 3: The material flow set F, F = {e_j | ∀i, j = 1,...,n}, is defined as a number of entity precedences, each of which is represented as a pair of supply chain entities.

Definition 4: A supplier entity is defined as e_j :=< c_i, op_j >. The supplier entity set E^S is defined as E^S :=< C, OP >, where C = {e_i | ∀i = 1,...,m} is a set of component to be fabricated by the set of suppliers E^S = {e_j | ∀i = 1,...,n}; OP = {op_j | ∀i = 1,...,n} is a set of operation attribute settings for processing c_i.

Definition 5: The component set C is described by a set of variety parameters, C = {VP_i | ∀i = 1,...,m}. A VP_i assumes n particular value instances associated with n components, and thus forms a set VP^*_i, VP^*_i = {vp^*_j | ∀j = 1,...,n}. A c_i is specified by a number of particular parameter values, c_i = [P^*_i]_{∀k = 1,...,m; j = 1,...,n}. Thus, each P^*_i is an instance of VP^*_i, and accordingly c_i ∈ [VP^*_1 × ⋯ × VP^*_k × ⋯ × VP^*_m].

Definition 6: The operation setting set op is defined as a tuple: op =< q, ld >, where q denotes the quantity of the
items to be produced by that operation; \( ld \) represents the lead time of the operation.

Definition 7: An assembly entity is defined as \( e_i^A = \langle c_i, C^A, op_i \rangle \), where \( c_i \) is the assembly produced by \( e_i^A \), \( C^A = \{c_1, c_2, \ldots, c_j\} \) is the set of components required by \( c_i \); \( op_i \) is the set of operation attribute settings.

Definition 8: A supply chain entity is defined by an attribute set \( a_i = \langle ct^i, iv^i \rangle \), where \( ct^i \) represents an operations cost; \( iv^i \) denotes the existing inventory of the entity. \( ct^i = \langle ct^i_1, ct^i_2, ct^i_3 \rangle \) contains three types of costs with respect to the production, inventory and transportation costs. \( iv^i = \langle iv^i_1, iv^i_2, iv^i_3 \rangle \) includes the inventory of raw materials, work-in-processes and finished products. Thus, the attribute set is further extended as \( a_i = \langle ct^i_1, ct^i_2, ct^i_3, iv^i_1, iv^i_2, iv^i_3 \rangle \).

Definition 9: A customer demand \( T \) has three types of features, \( T = \langle f^{Pd}, f^{Ps}, f^S \rangle \). The product feature \( f^{Pd} \) determines the product structure, i.e., \( C = \{c_i \mid \forall i = 1, \ldots, m\} \). The process feature \( f^{Ps} \) specifies the operation parameters, such as quantity \( q_i \) and lead time \( lt_i \). The supply chain feature \( f^S \) describes the entity attributes with respect to cost \( ct^i \) and inventory \( iv^i \).

![Figure 1. Elements and relationships in supply chain configuration.](image)

Fig. 1 shows the correlations among elements in the various domains of a supply chain network. Customer demands are transformed to the specifications on products, processes and supply chains. Entities in a supply chain configuration and attribute settings are determined by the product and process variants. The variations in product and process are transferred and reflected on the supply chain structures.

III. CPN MODELING FORMALISM

A supply chain model is built using a set of objects and gates. Each place corresponds to a place in a PN model and is denoted by a circle. It can represent various entities in a supply chain system, e.g., suppliers, manufacturers. Each node has an attribute set \( C = \{c_1, c_2, \ldots, c_j\} \), where \( c_i \) is associated with an attribute, such as amount, lead time and cost.

- \( E \) Place node set \( E = \{e_1, e_2, \ldots\} \).
- \( C \) Attribute set of place node \( c_i \), \( C = \{c_1, c_2, \ldots\} \).

Gates are denoted by bars with attached function sets. These functions evaluate and select the entities in the output place nodes when the gates are fired. In other words, the functions determine the material flows and select the suitable suppliers.

- \( G \) Gate set \( G = \{g_1, g_2, \ldots\} \).
- \( F \) Function set of gate \( g_i \), \( F = \{f_1^1, f_2^1, \ldots, f_r^1\} \).
- \( f_r^1 \) Function of gate \( g_i \).

For a flow \( p_i \rightarrow g_j \) (from place node \( p_i \) to gate \( g_j \)), \( p_i \) is called an input place node. For a gate \( g_j \), all the input place nodes consist of a node subset: the input place subset \( I^j \). If a flow is from gate \( g_j \) to node \( p_i \) (\( g_j \rightarrow p_i \)), \( p_i \) is called an output place node. For a gate \( g_j \), all the output place nodes consist of a place node subset: the output place subset \( O^j \).

- \( I^j \) Input place node set of gate \( g_j \).
- \( O^j \) Output place node set of gate \( g_j \).
- \( P^j \) Subset of place node set of gate \( g_j \), \( P^j = I^j \cup O^j \).

A configuration of supply chain entities determines the interconnections among them, be they are suppliers, manufacturing plants, logistics companies, final assembly plants, packaging centers, and transshipment points. Determined by the product specifications and the particular customer demands, different supply chain configurations can be derived from a generic network and they may not have the same structures. In general, there are four types of basic supply chain structures, including a serial, convergent, divergent, and network structures.

The attributes attached to place nodes convey important information pertaining to specific entities in a supply chain. In the modeling formalism, the static model of a generic supply chain network contains multiple nodes representing supply chain entities. Each of the entities has its specific capabilities in terms of manufacturing capacity, delivery time, and inventory. Corresponding to a certain customer order, several specific models, each of which is associated with a feasible supply chain configuration, can be constructed. The gates in the static model of the generic supply chain network control the selection of qualified suppliers so as to form specific supply chains in response to different customer orders. The functions associated with each gate evaluate each supplier’s capability and specify the best fit-one.
IV. APPLICATION OF CPN SCC MODEL

The proposed modeling formalism has been applied to a global company XYZ (disguised name) that produces motors (MTs). For illustrative simplicity, the data used are simplified version of the original data.

A motor typically has four base components and two assemblies, including a rotor (Rt), a stator (St), a base (Bs), a shield (Sh), a drive assembly (DA), and a case assembly (CA). Accordingly, four types of suppliers are involved in motors’ production: Rotor supplier (SRt) for fabricating Rt, Stator supplier (SSh) for St, Base supplier (SBs) for Bs, and Shield supplier (Sh) for Sh. A final MT is produced by assembly operations performed at three assembly entities, namely Amt for CA assembly, ADa for DA assembly and AMT for assembling the final MTs.

Fig. 2 shows the generic supply chain network of XYZ. For each material type, may it need fabrication or assembly operations, several capable entities compete with one another to be included in a particular supply chain. Furthermore, various operations parameters, such as quantity, delivery time and cost, are used to represent the entities with respect to different product variants.

The rationality of configuring supply chains for delivering different customer orders originates from the fact that multiple suppliers are able to provide the same materials with different conditions and competency. In order to validate variant derivation, a set of constraints, i.e., restrictions on resources, manufacturing capabilities and financial performance, should be satisfied. From the generic supply chain model, a number of possible supply alternatives for a specific product can be derived based on the set of parameters that describe the product and customer order.

Among all product variants of the MT family, we choose two for supply chain modeling: MTPr associated with supply chain SC1 and MTPc corresponding to supply chain SC2. Table I lists the set of specific items involved in MTPr and MTPc along with their parameters. For illustrative simplicity, not all the parameters that describe these items are shown in the table.

Fig. 3 gives the static CPN model of the supply chain network of XYZ. The set of objects involved in the model produce Rt, Sh, St, Bs, Sh, CA, DA, and MT. Rather than specific entities, these objects are generic items and entail sets of similar supply entities of same types. Each entity is specified by its unique attribute values. The set of gates, including g1, g2, g3 and g4, indicate the occurrence of certain events. In the model, g1 is placed between place P1 (recording customer orders) and the set of places P2 (representing supply chain entities) so as to allocate the tasks to the selected entities. Gate g2 is located after the final assembly AMT and prior to two assembly groups, ADa and Aca. It decomposes the task of producing MT to corresponding assembly operations for producing two assemblies: DA and CA. To further decompose the two assemblies into components: Rt, St, Bs, and Sh, gates g3 and g4 are introduced to select the suitable supplier entities.

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Table I. Entity and attribute elements in the CPN model in Figure 3.

<table>
<thead>
<tr>
<th>Group Node</th>
<th>Entity Node</th>
<th>ITEM</th>
<th>Process Attribute</th>
<th>Supply Attribute</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>Quantity</td>
<td>Lead Time</td>
</tr>
<tr>
<td>P2</td>
<td>P21 Mt Co. in Vaasa</td>
<td>MT1</td>
<td>p11_o</td>
<td>p11_i</td>
</tr>
<tr>
<td>P2</td>
<td>P22 Mt Co. in Germany</td>
<td>MT2</td>
<td>p22_o</td>
<td>p22_i</td>
</tr>
<tr>
<td>P2</td>
<td>P23 Mt Co. in Helsinki</td>
<td>MT3</td>
<td>p23_o</td>
<td>p23_i</td>
</tr>
<tr>
<td>P3</td>
<td>P31 Drive Assembly 1</td>
<td>DA1</td>
<td>p31_o</td>
<td>p31_i</td>
</tr>
<tr>
<td>P3</td>
<td>P32 Drive Assembly 2</td>
<td>DA2</td>
<td>p32_o</td>
<td>p32_i</td>
</tr>
<tr>
<td>P4</td>
<td>P41 Case Assembly 1</td>
<td>CA1</td>
<td>p41_o</td>
<td>p41_i</td>
</tr>
<tr>
<td>P4</td>
<td>P42 Case Assembly 2</td>
<td>CA2</td>
<td>p42_o</td>
<td>p42_i</td>
</tr>
<tr>
<td>P5</td>
<td>P51 Vaasa Rt Co</td>
<td>Rt1</td>
<td>p51_o</td>
<td>p51_i</td>
</tr>
<tr>
<td>P5</td>
<td>P52 German Rt Co</td>
<td>Rt2</td>
<td>p52_o</td>
<td>p52_i</td>
</tr>
<tr>
<td>P6</td>
<td>P61 India St. Co</td>
<td>St1</td>
<td>p61_o</td>
<td>p61_i</td>
</tr>
<tr>
<td>P7</td>
<td>P71 Vaasa Ca Co</td>
<td>Bs1</td>
<td>p71_o</td>
<td>p71_i</td>
</tr>
<tr>
<td>P7</td>
<td>P72 Helsinki Ca Co</td>
<td>Bs2</td>
<td>p72_o</td>
<td>p72_i</td>
</tr>
<tr>
<td>P8</td>
<td>P81 Oulu Sh Co</td>
<td>Sh1</td>
<td>p81_o</td>
<td>p81_i</td>
</tr>
<tr>
<td>P8</td>
<td>P82 Helsinki Ca Co</td>
<td>Sh2</td>
<td>p82_o</td>
<td>p82_i</td>
</tr>
</tbody>
</table>
As shown in Fig. 3, each supply chain entity $P_i$ might represent either a specific company that provides a certain item, or a group of companies, $\{P_{ij} \mid j \in 1,m\}$, that can produce the same items. Each entity is represented by two process parameters, $P^{ij}_i$ and $P^{ij}_j$, and supply parameters, $P^{ij}_s$ and $P^{ij}_a$. Parameter $P^{ij}_i$ indicates the material quantity allocated to the entity $P_{ij}$, $P^{ij}_j$ represents the delivery time required for the provided parts or components. $P^{ij}_s$ stands for the cost incurred in production. It can be further decomposed to a production cost, an inventory cost and a transportation cost. $P^{ij}_a$ refers to the inventory of the entity.

The generic supply chain model can respond to diverse product structures, process settings and supply parameters, which are specified in different customer orders. Configuring a supply chain refers to the process of determining specific entities from a generic model in response to a particular customer order. It includes 1) the derivation of place nodes ($P_i \rightarrow P_{ij}$) and the precedence relationships of entities ($P_m \sim P_n$); and 2) the determination of entity parameters, e.g., quantity $P^{ij}_i V^*_1$, delivery time $P^{ij}_j V^*_1$ and cost $P^{ij}_s V^*_1$. As a result, for each generic node $P_i$, the configuration obtains a five tuple: $<P_{ij},P^{ij}_i,P^{ij}_j,P^{ij}_s,P_{ij}>P_{mn}$. The configuration process is guaranteed through the firing of gates with respect to colored tokens. Based on the functions and algorithms $f^j_1, f^j_2, \ldots, f^j_m$ attached to the gate $g_j$, tokens move from an upstream place node $P_m$ to a downstream place node $P_n$ so as to select the right entity $P_{ij}$. The control logic determines the node parameter values. For example, give a $P_2$ variant that is specified as $<P_{22},200,15,45>$, we need to find the downstream node $P_{3i}$ and determine values of parameters $P^{ji}_1$, $P^{ji}_2$ and $P^{ji}_3$. The function set associated with $g_2$ is $(g^2_1,g^2_2,g^2_3,g^2_4)$:

$g^2_1 : \text{if } (P^{ji}_1 V^*_1 - P^{ji}_2 V^*_1) t_{23} > P^{ji}_3, \text{ then } P^{ji}_3 = \phi$

$g^2_2 : \text{if } P^{ji}_2 V^*_1 < P^{ji}_3, \text{ then } P^{ji}_3 = \phi$

$g^2_3 : \text{if } P^{ji}_2 V^*_1 > P^{ji}_3, \text{ then } P^{ji}_3 = \text{min } mize(P^{ji}_3)$

$g^2_4 : \text{if } P^{ji}_2 V^*_1 > P^{ji}_3, \text{ then } P^{ji}_3 = \text{min } mize(P^{ji}_3)$

The function $g^2_1$ specifies that $P_{3i}$ cannot be selected to proceed to $P_{2j}$ under that condition that its production capacity is lower than the allocated quantity. The allocated quantity of $P_3$ can be obtained by 1) subtracting its existing inventory from the quantity of $P_{2j}$, and 2) timing the bill of materials index $t_{23}$. Index $t_{23}$ represents the number of unit parts provided by $P_3$ for one component in $P_2$.

The function $g^2_2$ ensures that the supplier should be capable of providing materials in time. For such entities that cannot meet the delivery time deadline of its upstream entity, they cannot be selected to be included in a supply chain.

The function $g^2_3$ shows the economic selection criteria, such that the entity with minimum cost will be selected to fulfill the customer orders.

The function $g^2_4$ imposes further constraints on the delivery, such that the suppliers with quick response will be selected first. Fig. 4 shows an example of a specific supply chain configuration obtained from the static model of the supply chain network of XYZ for motor $MT^*_i$.

Based on the semantics of Petri-nets and a structural change handling mechanism discussed in [5], the model $SCN^i_{MT}$ is formally described as follows.

$SCN^i_{MT} = (C_i, O_i, P_i, R_i, L_i)$

$C_i = (R_i, St_i, DA_i, CA_i, MT_i)$

Figure 4. A supply chain configuration for $MT^*_i$. 

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O_i = \{ M, A_{MT_1}, A_{DA_1}, A_{CA_1}, S_{R_1}, S_{A_1}, S_{B_1}, S_{D_1} \} 

P_i = \left\{ \begin{array}{l} (P_i^{MT_1} \land P_i^{DA_1} \land P_i^{CA_1} \land P_i^{A_{MT_1}}) \\
(P_i^{MT_1} \land P_i^{DA_1} \land P_i^{A_{MT_1}}) \\
(P_i^{MT_1} \land P_i^{DA_1} \land P_i^{A_{MT_1}}) \\
(P_i^{MT_1} \land P_i^{DA_1} \land P_i^{A_{MT_1}}) \end{array} \right\}

R_i = \left\{ \begin{array}{l} R_{MT_1}^{A_{MT_1}}, R_{MT_1}^{A_{MT_1}} \\
R_{DA_1}^{A_{MT_1}}, \ldots \\
R_{DA_1}^{A_{MT_1}}, \ldots \\
R_{DA_1}^{A_{MT_1}}, \ldots \end{array} \right\}

SC_{V_{1}^{MT_1}} \text{ stands for the specific supply chain configuration}

to produce \( MT_1 \) for the customer order. \( M = C, O, P, R, \) and \( L \)
represent five sets of items, entities, parameters, message relationships, and entity precedence, respectively.

As shown in Fig. 4, the involved objects in current system include \( M, A_{MT_1}, A_{DA_1}, A_{CA_1}, A_{RL_1}, A_{SL_1}, A_{BS_1}, \) and \( A_{SA_1} \). \( M \)
represents a set of customer orders. Each customer might specify different product specifications and require various
operations and supply parameter values in terms of time, cost, and quantity. Thus, each M instance requires a specific supply
chain. A particular order is transferred to motor assembly plant after gate \( g_1 \) is fired. Each entity receives only one
token. The data attached in tokens, \( P_1^{MT_1} \land P_2^{MT_1} \land P_3^{MT_1} \land P_4^{MT_1} \), are the set of product and process parameter values associated with items. The logic function of
\( g_1 \) specifies the token flow, according to which the assemblies are selected, as well as the corresponding quantity,
lead time and cost. In \( A_{MT_1} \), the color of the token represents
product \( MT_1 \) to be produced. The parameters
\( P_1^{MT_1} \land P_2^{MT_1} \land P_3^{MT_1} \land P_4^{MT_1} \) reflect the process requirements of
assembly company \( A_{MT_1} \), such as quantity, delivery time and cost. When the token moves from \( A_{MT_1} \) to \( A_{DA_1} \), the set of parameters
\( P_1^{MT_1} \land P_2^{MT_1} \land P_3^{MT_1} \land P_4^{MT_1} \) is assigned to \( A_{DA_1} \) so
as to select the suitable DA. According to the function of gate \( g_2 \), \( A_{DA_1} \) is selected and the corresponding parameters are
calculated and allocated.

After assembly components DA and CA have been allocated to the two entities of \( A_{DA_1} \) and \( A_{CA_1} \), the
configuration of supply chain is decomposed into two threads and two tokens are generated accordingly. Through the firing
of gate \( g_3 \), the production parameters
\( P_1^{MT_1} \land P_2^{MT_1} \land P_3^{MT_1} \land P_4^{MT_1} \) are further transferred to supplier
\( S_{R_1} \). The operation parameters
\( P_1^{MT_1} \land P_2^{MT_1} \land P_3^{MT_1} \land P_4^{MT_1} \)
are determined according to the capacity and inventory of the
assembly plant \( A_{DA_1} \) as well. Similarly, supplier \( S_{SA_1} \)
receives the set of parameters
\( P_1^{MT_1} \land P_2^{MT_1} \land P_3^{MT_1} \land P_4^{MT_1} \).

Upon the configuration of DA and CA assemblies, two new tokens representing the selected assembly DA and CA are
generated. The colors of tokens have been changed subsequently. A new color \( f \) is assigned to DA and color \( g \)
is assigned to CA, whilst the original color \( d \) of MT has been
removed. There are two types of relations between entities, \( OR \) and \( AND \). An \( OR \) relation means that two entities can be selected separately. An \( AND \) relation indicates that two
entities should be selected together, such that their parameters are correlated and to be calculated jointly. Once the gate with
an \( OR \) relation is fired, the token will not be changed. When the gate with an \( AND \) relation is fired, the token will be removed and two new color tokens are generated.

V. CONCLUSION

Facing intense global competition, manufacturers have been striving to improve their system performance through
configuring proper supply chains in respond to customer requirement changes. There is a need for a decision support
tool for supply chain configuration modeling with joint consideration of products, processes and logistics design.
This paper discusses a set of formalisms based on color Petri-net attempting to model and evaluate supply chain
configurations. The formalism represents the operations and design of a supply chain as an abstracted network, where the
important information is documented as the attributes of the corresponding entities. Based on the develop formalism, a
manager can model different supply chain operations and design decisions and further use the results to make decisions
about supply chain reengineering, as well as product and process design.

REFERENCES